...sociated with magnetic anisotropy, dispersion hardening, ductile–brittle transition of refractory metals and solution hardening in alloy single crystals.

If desired, the target of the x-ray tube may consist of an alloy, in which case the diffraction pattern recorded on the film will consist of ellipses characteristic of the elements of the target material.[1] Consequently, precision measurements can be carried out as a function of different wavelengths under identical photographic processing conditions. If the alloy of the target material contains elements of high and low atomic number, strain measurements can be carried out on the specimen which may be made a function of the depth of penetration of the corresponding radiation. This aspect may turn out to be particularly useful for a number of problems in physical metallurgy, e.g., in studies of internal oxidation, case hardening, and dispersion hardening.

It should be borne in mind, however, that there are a number of factors which limit the stress–strain analysis of materials by the divergent beam method and these will now be discussed.

(1) The method in its present form is solely restricted to the study of single crystals having faces not less than 4 mm². It is quite possible, however, that in the future it can be extended to polycrystalline specimens if the following modifications in instrumentation are adopted: (a) construction of an x-ray tube with spot size of about 1 μ; (b) narrowing of the tip of the x-ray tube so as to reduce the instrumental obstruction to the x-rays diffracted by the specimen. If these requirements are fulfilled, the specimen may be brought closer to the x-ray source and grains approximately 250 μ in size can be studied.

(2) The crystal must exhibit in its undeformed state a fairly high degree of lattice perfection and must be free of surface distortions. The undeformed state is regarded as the reference state to which all measured strains, that is, $\Delta d/d$ values, are referred. The required degree of lattice perfection for the reference state is defined by the absence of those characteristics of the x-ray pattern discussed in Sec. 4.

If the strain distribution in the crystal becomes inhomogeneous and the pattern shows manifestations of broadening, kinking, and displacement of the lines, the strain analysis has to be augmented by a Fourier transform analysis of the line profile (Sec. 4). The divergent beam method offers, however, the great advantage that the individual (hkl) reflections of a form are not superimposed as in the powder technique, which is the basis of current Fourier methods.

(3) More than six independent (hkl) reflections have to be analyzed.[1] Furthermore, to obtain a representative sampling of the strain ellipsoid the crystal must be studied in different directions. This necessitates irradiation of a number of different plane faces of the specimen.

(4) The smallest measurable strains for a particular (hkl) reflection must be larger than the experimental error defined by the corresponding $\bar{\sigma}_d$ value (Sec. 7).

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APPENDIX A

Least-Squares Determination of $y=mx+B$

The least-squares estimates of m_i, $i=1$, 2 and B_i, $i=1$, 2 are obtained by minimizing the sum of the squares of the deviations:

$$S=\sum_j (y_{ij}-m_i x_{ij}-B_i)^2, \quad i=1,2; \quad j=1, n_i,$$

where n_i is the number of determinations for each line.

We have, using the expressions from the regression theory,[11,12]

$$m_i=\sum_j y_{ij}(x_{ij}-\bar{x}_i)/\sum(x_{ij}-\bar{x}_i)^2, \quad i=1, 2 \quad \text{(A1)}$$

$$B_i=\bar{y}_i-m_i\bar{x}_i, \quad i=1, 2. \quad \text{(A2)}$$

The variance of estimate is given by

$$V_i(y/x)=\sum(y_{ij}-m_i x_{ij}-B_i)^2/(n-2), \quad i=1, 2, \quad \text{(A3)}$$

where m_j and B_j are given by (A1) and (A2), respectively, and the notation $V_i(y/x)$ stands for the variance of y given x, that is, the variance about the least-squares line.

The variance of the slope is given by

$$V(m_i)=V_i(y/x)/\sum_j(x_{ij}-\bar{x}_i)^2, \quad i=1, 2. \quad \text{(A4)}$$

d Spacing and its Standard Error

α and d are given by Eqs. (5) and (6). Knowing the standard error of the slopes m_1 and m_2 we can compute the standard error of the d spacing. In fact, we have

$$V(\alpha)=\frac{k^2}{4}\left[\frac{V(m_1)}{[1+(km_1)^2]^2}+\frac{V(m_2)}{[1+(km_2)^2]^2}\right] \quad \text{(A5)}$$

and

$$V(d)=(\lambda^2/4)(\sin^2\alpha/\cos^4\alpha)V(\alpha). \quad \text{(A6)}$$

The standard error of d is then

$$\sigma_d=[V(d)]^{\frac{1}{2}}. \quad \text{(A7)}$$

Weighted Least Squares

The relation between the lattice parameter a' and x,

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[11] A. Hald, *Statistical Theory with Engineering Applications* (John Wiley & Sons, Inc., New York, 1952), Chap. 18.

[12] A. H. Bowker and G. J. Lieberman, *Engineering Statistics* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1959), Chap. IX.